

交通大學 應數系 統計學 習題八

日期:2014.01.09 時間:6:30 教室:SA214

一. 回答時盡可能詳細、清楚，若有使用到的定理，可直接引述該定理名稱。

二. 主題以外的內容當作已知，不必多做繁瑣的證明。

1. Consider a random sample $X_1 \dots X_n$ from a two-parameter exponential distribution with p.d.f.

$$f_X(x; \eta) = e^{-(x-\eta)}, \quad x > \eta$$

(a) Show that the statistic $Y_1 = \min\{X_1 \dots X_n\}$ is sufficient for η

(b) Show that the statistic $Y_1 = \min\{X_1 \dots X_n\}$ is complete

(c) Verify that $Y_1 - 1/n$ is the UMVUE of η

2. Consider a random sample of size n from a distribution with pdf

$$f(x; \theta) = \begin{cases} \frac{(\ln \theta)^x}{\theta x!} & x=0, 1, \dots; \theta > 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find a complete sufficient statistic for θ

(b) Find the UMVUE of $\ln \theta$

(c) Find the UMVUE of $(\ln \theta)^2$

3. Let $X_1 \dots X_n$ be a random sample from a Binomial distribution, $X_i \sim b(1, p)$; $0 < p < 1$

(a) Find the UMVUE of $\text{Var}(X) = p(1-p)$

(b) Find the UMVUE of p^2

4. Consider a random sample of size n from a distribution with pdf $f(x; \theta) = \theta x^{\theta-1}$ if $0 < x < 1$ and zero otherwise; $\theta > 0$

(a) Find the UMVUE of $1/\theta$

(b) Find the UMVUE of θ

5. Consider a random sample $X_1 \dots X_n$ from a Poisson distribution, $X_i \sim \text{Poisson}(\mu)$; $\mu > 0$.

Find the UMVUE of $P[X=0] = e^{-\mu}$.