交通大學 應數系 統計學 練習三

日期:2014.10.30 時間:5:30 教室:SA214

一. 回答時盡可能詳細、清楚，若有使用到的定理，可直接引述該定理名稱。

二. 主題以外的內容當作已知，不必多做繁瑣的證明。

1. Define the covariance of X and Y, denoted it by Cov(X,Y)=E[(X-E(X))(Y-E(Y))]. Show that Var(X+Y)=Var(X) +2Cov(X,Y) +Var(Y).

2. If $X \sim N(\mu, \sigma^2)$, show that $E(X^3) = \mu^3 + 3\sigma^2\mu$.

3. Let X, Y be i.i.d. standard normal random variable, derive the distribution of X/Y.

4. If $X \sim N(\mu_X, \sigma_X^2)$, $Y \sim N(\mu_Y, \sigma_Y^2)$ and $X \coprod Y$, show that $X-Y \sim N(\mu_X - \mu_Y, \sigma_X^2 + \sigma_Y^2)$.

5. If $X_1, \cdots, X_n$ is a random sample from $\text{Exp}(\beta)$, please derive the distribution of $Y = \sum_{i=1}^{n} X_i$. Prove it by mgf method.

6. If $X \sim \text{Gamma}(\alpha, \beta)$, show that $aX \sim \text{Gamma}(\alpha, a\beta)$.

7. If $X_1, \cdots, X_n$ is a random sample from $\text{Gamma}(0.5, \beta)$, please derive the distribution of variable $Y = \sum_{i=1}^{n} \frac{2X_i}{\beta}$.

8. There are three random variables X, Y and Z with joint m.g.f. as $M(t_1, t_2, t_3) = (0.3 + 0.7e^{t_1})^8 e^{3t_3 + t_2^2 + \frac{1}{2}t_3^2}$. Are these random variables independent?