交通大學 應數系 統計學 練習四

日期:2014.12.04　時間:5:30　　教室:SA214

一. 回答時盡可能詳細、清楚，若有使用到的定理，可直接引述該定理名稱。

二. 主題以外的內容當作已知，不必多做繁瑣的證明。

1. Let $X_1, X_2, \cdots, X_n$ be r.v.'s with p.d.f. $f_n(x) = \begin{cases} 1-\frac{1}{2^n}, x=0 \\ \frac{1}{2^n} \quad ,x=1 \\ 0 \quad ,o.w \end{cases}$, show that $X_n \xrightarrow{p} 0$.

2. If $Y_1, \cdots, Y_n$ is a random sample of size n from a normal population with mean μ and variance σ^2. Assuming $n=2k$ for some integer k, one possible estimator for σ^2 is given by: $\hat{\sigma}^2 = \frac{1}{2k}\sum_{i=1}^{k}\left(Y_{2i} - Y_{2i-1}\right)^2$.

   a. Show that $\hat{\sigma}^2$ an unbiased estimator for σ^2.
   b. Show that $\hat{\sigma}^2$ is a consistent estimator for σ^2.

3. If $U_n \xrightarrow{p} u$ and $g(x)$ is continuous at $x = u$, show that $g(U_n) \xrightarrow{p} g(u)$.

4. Let $Y_1, \cdots, Y_n$ be i.i.d. random variables, with pdf $f(y) = \begin{cases} (\theta+1)y^\theta, \ 0 \le y \le 1 \\ 0, \quad \text{elsewhere.} \end{cases}$.

   Is $\frac{2\overline{Y}-1}{1-\overline{Y}}$ consistent for θ ?

5. Use the Chebyshev's Inequality to prove Weak Law of Large Numbers directly.

6. Let $X_1, \cdots, X_n$ is a random sample from $f(x;\theta) = \frac{2\theta^2}{x^3}, 0 \le \theta \le x$, find the M.M.E. $\hat{\theta}$ of θ.

   Is $\hat{\theta}$ the consistent estimator?