交通大學 應數系 統計學 練習五

日期:2014.12.11　時間:5:30　　教室:SA214

一. 回答時盡可能詳細、清楚，若有使用到的定理，可直接引述該定理名稱。

二. 主題以外的內容當作已知，不必多做繁瑣的證明。

1. Let $X_1,\cdots,X_n$ is a random sample from Pareto distribution, i.e. f(x;θ)=$\dfrac{\theta}{x^2}$, $0<\theta\le x<\infty$.
   Does the M.M.E. of θ exist? If yes, what is it? If no, find the MLE of θ.

2. If $X_1,X_2,\cdots,X_n$ be a random sample from U(0,1). Show that $X_{(1)}$~Beta(1,n) and $X_{(n)}$~Beta(n,1), where $X_{(1)}$ and $X_{(n)}$ is the smallest and the largest order statistic, respectively.

3. If $X_1,X_2,\cdots,X_n$ be a random sample from $U(0,2\theta)$, find the MLE of θ. Is it an unbiased estimator of θ?

4. $X_1,\ldots,X_n$ is a random sample from U($\theta-0.5,\theta+0.5$).
   a. Find the MLE $\hat{\theta}$ for θ. Is the MLE unique? If not, give the set of MLE's.
   b. Show that $\hat{\theta}=0.5(\mathrm{X}_{(1)}+\mathrm{X}_{(n)})$ is an unbiased estimator of θ,
   where $\mathrm{X}_{(1)}=\min \mathrm{X}_i$, $\mathrm{X}_{(n)}=\max \mathrm{X}_i$. Is it a MLE?

5. Let $X_1,\ldots,X_n$ be a random sample from U(θ_1,θ_2), $\theta_1<\theta_2$. Find the MLE of θ_1 and θ_2.

6. Let $Y_1,\cdots,Y_n$ be independent, Y_i is Poisson distributed with mean λx_i, where x_i's are known. Find the MLE of λ.

7. $X_1,\ldots,X_n$ is a random sample from $f(x)=\dfrac{1}{\sigma}e^{\frac{-(x-\mu)}{\sigma}}$, $x\ge\mu$, $\mu\in R$, $\sigma>0$.
   a. Find the MLE for μ and σ.
   b. Find the MLE for $P(X\ge t)$ where $t>\mu$.