

交通大學 應數系 統計學 練習六

日期:2013.12.25 時間:5:30 教室:SA214

一. 回答時盡可能詳細、清楚，若有使用到的定理，可直接引述該定理名稱。

二. 主題以外的內容當作已知，不必多做繁瑣的證明。

1. X_1, \dots, X_n is a random sample from $N(\theta, 1)$.
 - a. Find the MLE $\hat{\theta}$ for θ .
 - b. Find the CRLB for the unbiased estimator of θ .
 - c. Is MLE the UMVUE?
2. X_1, \dots, X_n is a random sample from $N(0, \sigma^2)$.
 - a. Find the MLE $\hat{\sigma}^2$ for σ^2 .
 - b. Find the CRLB for the unbiased estimator of σ^2 .
 - c. Is MLE the UMVUE?
3. Y_1, \dots, Y_n is a random sample from an exponentially distribution with
p.d.f $f(y) = \frac{1}{\theta} e^{-y/\theta}, y > 0, \theta > 0$.
 - a. Find the MLE $\hat{\theta}$ for θ .
 - b. Find the CRLB for the unbiased estimator of θ .
 - c. Is MLE $\hat{\theta}$ the UMVUE?
 - d. Find the MLE $\hat{\eta}$ of the population variance η .
4. Let β_1, \dots, β_3 be the angles for a triangle in degrees, so $\beta_1 + \beta_2 + \beta_3 = 180$; and let Y_1, \dots, Y_3 be measurements of these angles. Assume $Y_i \sim N(\beta_i, \sigma^2), i=1,2,3$.
 - a. Find the MLE for β_1 and β_2 .
 - b. Find a unbiased estimator of σ^2 .
5. Let X_1, \dots, X_n is a random sample from $\text{Exp}(\theta)$.
 - a. Use the Factorization Theorem to find a sufficient statistic of θ .
 - b. Show that it is a sufficient statistic of θ by using definition.
6. Let (X, Y) have joint density function $f(x, y) = 2, 0 < x < y < 1$.
 - a. Find $E(X)$ and $\text{Var}(X)$.
 - b. Verify $E[E(X|Y)] = E(X)$ and $\text{Var}[E(X|Y)] + E[\text{Var}(X|Y)] = \text{Var}(X)$.
 - c. Are X and Y independent?