交通大學 應數系 統計學 練習七

日期:2015.01.05　時間:1:20　　教室:A302

一. 回答時盡可能詳細、清楚，若有使用到的定理，可直接引述該定理名稱。

二. 主題以外的內容當作已知，不必多做繁瑣的證明。

1. Consider a random sample $X_1 ... X_n$ from a two-parameter exponential distribution with p.d.f. $f_X(x;\eta)=e^{-(x-\eta)}$, $x>\eta$

(a) Show that the statistic $Y_1=\min\{X_1 ... X_n\}$ is sufficient for η

(b) Show that the statistic $Y_1=\min\{X_1 ... X_n\}$ is complete

(c) Verify that Y_1-1/n is the UMVUE of η

2. Consider a random sample of size n from a distribution with pdf

$$f(x;\theta)=\begin{cases}\dfrac{(\ln\theta)^x}{\theta x!} & x=0,1...;\theta>1\\ 0 & \text{otherwise}\end{cases}$$

(a) Find a complete sufficient statistic for θ

(b) Find the UMVUE of $\ln\theta$

(c) Find the UMVUE of $(\ln\theta)^2$

3. Let $X_1 ... X_n$ be a random sample from a Binomial distribution, $X_i \sim b(1,p)$; $0<p<1$

(a) Fine the UMVUE of Var(X)=p(1-p)

(b) Find the UMVUE of p^2

4. Consider a random sample of size n from a distribution with pdf $f(x;\theta)=\theta x^{\theta-1}$ if $0<x<1$ and zero otherwise; $\theta>0$

(a) Find the UMVUE of $1/\theta$

(b) Find the UMVUE of θ

5. Consider a random sample $X_1 ... X_n$ from a Poisson distribution, $X_i \sim$ Poisson(μ) ;$\mu>0$. Find the UMVUE of $P[X=0]=e^{-\mu}$.

6. Suppose $X_1,\ldots,X_n$ be a random sample from a normal distribution with mean μ and variance 1.

(a) Find the UMVUE of μ^2.

(b) Does the variance of the UMVUE you obtained in (a) reach the Cramer-Rao lower bound?

7. Let $Y_1,\ldots,Y_n$ be a random sample from a distribution with a denstiy function

$f(y;\theta)=\dfrac{2y}{\theta^2}$, $0<y<\theta$

(a) Is $\hat{\theta}=\dfrac{3}{2}\overline{Y}$ an unbiased estimator of θ?

(b) Is the variance of $\hat{\theta}$ less than the Cramer-Rao lower bound for $f(y;\theta)$?