交通大學 應數系 統計學 習題一

演習課日期:2013.10.24　　時間:6:30　　上課教室:SA214

一. 回答時盡可能詳細、清楚，若有使用到的定理，可直接引述該定理名稱。

二. 主題以外的內容當作已知，不必多做繁瑣的證明。

1. Let X be a U(0,1) random variable. Let $Y=X^2$, find the p.d.f. of Y.

2. Let X have the uniform pdf $f_X(x)=\frac{1}{\pi},\ -\frac{\pi}{2}<x<\frac{\pi}{2}$. Find the pdf of $Y=\tan X$.

3. Suppose X has density $f(x)=\frac{1}{4},\ -1\le x\le 3$, and let $U=X^2$.
   (1) Find the distribution funtion $F(x)=P(X\le x)$ of U.
   (2) Find the density funtion of U.

4. Suppose X has p.d.f. with $f(x)=\begin{cases}\frac{3x^2}{a^3}, & 0<x<a\\ 0, & o.w\end{cases}$
   (1) If $P(X>1)=7/8$，find the value of a.
   (2) Find E(X).

5. Let the p.d.f of X be $f(x)=\begin{cases}\frac{1}{5}, x=1\\ \frac{2}{5}, x=2,4.\\ 0, o.w.\end{cases}$

   Find m.g.f. of X, E(X), Var(X).

6. Let Z be a standard normal random variable. Use the method of moment-generating function to find the probability distribution of Z^2.

7. Let X, Y be independent standard normal random variable, derive the distribution of X/Y.

8. Let Y_1 and Y_2 be i.i.d. exponential random variables, with pdf $f(x)=\lambda e^{-\lambda x}$, $x>0$. Find the density function of $U=\frac{Y_1}{Y_1+Y_2}$.