交通大學 應數系 統計學 習題二

日期:2013.10.31　時間:6:30　　教室:SA214

一. 回答時盡可能詳細、清楚，若有使用到的定理，可直接引述該定理名稱。

二. 主題以外的內容當作已知，不必多做繁瑣的證明。

1. If X has normal distribution $\mathrm{N}\left(\mu,\sigma^2\right)$, please derive the distribution of $Y=aX+b$.

2. If X~Gamma$(\alpha,\ \beta)$, show that $E(X^n)=\dfrac{\Gamma(\alpha+n)}{\Gamma(\alpha)}\beta^n$

3. Let X be a random variable such that $E(X-b)^2$ exists for all real b. Please show that $E(X-b)^2$ is a minimum when b=E(X).

4. Show that X~ Bernoulli(p) if and only if $E(X^n)=p$, n=1,2,...

5. If X,Y are independent, show that $\mathrm{E}\left[g(X)h(Y)\right]=\mathrm{E}\left(g(X)\right)\mathrm{E}\left(h(Y)\right)$.

6. If $X\sim\mathrm{N}\left(\mu_X,\ \sigma_X^2\right)$, $Y\sim\mathrm{N}\left(\mu_Y,\ \sigma_Y^2\right)$ and they are independent, show that $Z=X+Y\sim\mathrm{N}(\mu_X+\mu_Y,\sigma_X^2+\sigma_Y^2)$

7. If $X_1,\cdots,X_n$ is a random sample from Poisson(λ), please derive the distribution of $Y=\sum_{i=1}^{n}X_i$. Prove it by mgf method.

8. If $Y_1,\ Y_2,\ Y_3$ is a random sample from an exponential distribution with density function
$$f(y)=\begin{cases}\dfrac{1}{\theta}e^{-y/\theta}, & y>0\\ 0, & \text{elsewhere.}\end{cases}$$
Consider the following five estimators of θ:
$$\hat{\theta}_1=Y_1,\ \hat{\theta}_2=\frac{Y_1+Y_2}{2},\ \hat{\theta}_3=\frac{Y_1+2Y_2}{3}.$$
Which of these estimators are unbiased?