

交通大學 應數系 統計學 習題三

日期:2013.11.07 時間:6:30 教室:SA214

一. 回答時盡可能詳細、清楚，若有使用到的定理，可直接引述該定理名稱。

二. 主題以外的內容當作已知，不必多做繁瑣的證明。

1. If X_1, \dots, X_n is a random sample from $\text{Exp}(\beta)$, please derive the distribution of

$$Y = \sum_{i=1}^n X_i. \text{ Prove it by mgf method.}$$

2. If $X \sim \text{Gamma}(\alpha, \beta)$, show that $aX \sim \text{Gamma}(\alpha, a\beta)$.

3. If X_1, \dots, X_n is a random sample from $N(\mu, \sigma^2)$, and we know that $E(S^2) = \sigma^2$, find $E(S)$.

4. Let X, Y be a random sample from $U(0,1)$. Use the method of distribution functions to find the probability density function for $U = X + Y$.

5. Show that the variance of Y/n , where Y has a binomial distribution with n trials and a success probability of p , has a maximum at $p=0.5$, for fixed n .

6. If two independent random variables X and Y , $X \sim \text{Gamma}(5, 2)$, $Y \sim \chi^2(10)$, is X and Y a random sample?

7. If X_1, \dots, X_{10} are independent r. v.'s with mean $\mu_i = i$ and variance $\sigma_i^2 = i^2$, $i = 1, 2, \dots, 10$. Please find the mean and variance of $Y = \sum_{i=1}^{10} iX_i$.

8. There are three random variables X, Y and Z with joint m. g. f. $M(t_1, t_2, t_3) = (0.3 + 0.7e^{t_1})^8 e^{3t_3 + t_2^2 + \frac{1}{2}t_3^2}$. Are these random variables independent?