交通大學 應數系 統計學 習題四

日期:2013.12.05 時間:6:30 教室:SA214

一. 回答時盡可能詳細、清楚，若有使用到的定理，可直接引述該定理名稱。

二. 主題以外的內容當作已知，不必多做繁瑣的證明。

1. We define the covariance of X and Y and denoted by Cov(X,Y)=E[(X-E(X))(Y-E(Y))]. Show that Var(X+Y)=Var(X) +2Cov(X,Y) +Var(Y).

2. If Z1,...,Zp are independent and $\delta \geqq 0$ with Z1~N(δ ,1) and Zj~N(0,1), j=2...p, then W=$Z1^2+\ldots+Zp^2$ has the non-central chi-square distribution with non-centrality parameter δ^2 and p degrees of freedom.
Find the mean and variance of the non-central chi-square distribution on p degrees of freedom with non-centrality parameter δ^2.

3. One observation, X, is taken from a N(0,σ^2) population. Find an unbiased estimator of σ^2.

4. Let $X_1, X_2, \cdots, X_n$ be r.v.'s with p.d.f. $f_n(x)=\begin{cases} 1-\dfrac{1}{2^n}, x=0 \\ \dfrac{1}{2^n} \quad ,x=1 \\ 0 \quad ,o.w \end{cases}$, show that $X_n \xrightarrow{p} 0$.

5. Let $Y_1,\cdots,Y_n$ be i.i.d. random variables, with pdf $f(y)=\begin{cases} 3y^2, & 0\le y\le 1 \\ 0, & \text{elsewhere.} \end{cases}$.

   Show that $\bar{Y}$ converges in probability to some constant, and find the constant.

6. If $Y_1,\cdots,Y_n$ is a random sample of size n from a normal population with mean μ and variance σ^2. Assuming n=2k for some integer k, one possible estimator for σ^2 is given by: $\hat{\sigma}^2=\dfrac{1}{2k}\sum_{i=1}^{k}\left(Y_{2i}-Y_{2i-1}\right)^2$.

   a. Show that $\hat{\sigma}^2$ an unbiased estimator for σ^2.
   b. Show that $\hat{\sigma}^2$ is a consistent estimator for σ^2.

7. If $U_n \xrightarrow{p} u$ and $g(x)$ is continuous at $x = u$, show that $g(U_n) \xrightarrow{p} g(u)$.

8. Let $Y_1, \cdots, Y_n$ be i.i.d. random variables, with pdf $f(y) = \begin{cases} (\theta+1)y^{\theta}, & 0 \le y \le 1 \\ 0, & \text{elsewhere.} \end{cases}$.

Is $\dfrac{2\overline{Y}-1}{1-\overline{Y}}$ consistent for θ ?