交通大學 應數系 統計學 習題五

日期:2013.12.12 時間:6:30 教室:SA214

一. 回答時盡可能詳細、清楚，若有使用到的定理，可直接引述該定理名稱。

二. 主題以外的內容當作已知，不必多做繁瑣的證明。

1. Let $X_1, X_2, \cdots, X_n$ be r.v.'s with p.d.f. $f_n(x) = \begin{cases} 1-\frac{1}{n}, x=0 \\ \frac{1}{n}, x=\sqrt{n} \\ 0, o.w \end{cases}$, show that $X_n \xrightarrow{p} 0$.

2. If $X_1, X_2, \cdots, X_n$ be a random sample from U(0,1). Let Y_1 be the smallest of $X_1, X_2, \cdots, X_n$, and Y_n be the largest of $X_1, X_2, \cdots, X_n$.
Show that Y_1~Beta(1,n) and Y_n~Beta(n,1).

3. If $X_1, X_2, \cdots, X_n$ be a random sample from U(0,2θ), find the M.L.E. of θ. Is it an unbiased estimator of θ ?

4. Let $X_1, \ldots, X_n$ be a random sample from $U(\theta_1, \theta_2)$, $\theta_1 < \theta_2$. Find the MLE of θ_1 and θ_2.

5. Let $Y_1, \cdots, Y_n$ be independent, Y_i Poisson distributed with mean λx_i, where x_i's are known. Find the MLE of λ.

6. Let $X_1, \ldots, X_n$ be a random sample from Bernoulli(θ), $0 < \theta < 0.5$, find the MLE of θ.