交通大學 應數系 統計學 習題六

日期:2013.12.19 時間:6:30 教室:SA214

一. 回答時盡可能詳細、清楚，若有使用到的定理，可直接引述該定理名稱。

二. 主題以外的內容當作已知，不必多做繁瑣的證明。

1. $X_1,\ldots,X_n$ is a random sample from $U(\theta-0.5,\theta+0.5)$.
   a. Find the MLE $\hat{\theta}$ for θ. Is MLE unique? If not, please give the set of MLE's.
   b. Show that $\hat{\theta}=0.5(X_{(1)}+X_{(n)})$ is an unbiased estimator of θ,
   where $X_{(1)}=\min X_i$, $X_{(n)}=\max X_i$. Is it a MLE?

2. $X_1,\ldots,X_n$ is a random sample from the p.d.f. $f(x)=\frac{1}{\sigma}e^{\frac{-(x-\mu)}{\sigma}}$, $x\geq\mu$, $\mu\in R$, $\sigma>0$.
   a. Find the MLE for μ and σ.
   b. Find the MLE for $P(X\geq t)$ where $t>\mu$.

3. $X_1,\ldots,X_n$ is a random sample from $N(\theta,1)$.
   a. Find the MLE $\hat{\theta}$ for θ.
   b. Find the CRLB for the unbiased estimator of θ.
   c. Is MLE the UMVUE?

4. $X_1,\ldots,X_n$ is a random sample from $N(0,\sigma^2)$.
   a. Find the MLE $\hat{\sigma}^2$ for σ^2.
   b. Find the CRLB for the unbiased estimator of σ^2.
   c. Is MLE the UMVUE?

5. $Y_1,\ldots,Y_n$ is a random sample from an exponentially distribution with p.d.f $f(y)=\frac{1}{\theta}e^{-y/\theta}$, $y>0$, $\theta>0$.
   a. Find the MLE $\hat{\theta}$ for θ.
   b. Find the CRLB for the unbiased estimator of θ.
   c. Is MLE $\hat{\theta}$ the UMVUE?
   d. Find the MLE $\hat{\eta}$ of the population variance θ^2.