

# 交通大學 應數系 統計學 習題七

日期:2013.12.26 時間:6:30 教室:SA214

一. 回答時盡可能詳細、清楚，若有使用到的定理，可直接引述該定理名稱。

二. 主題以外的內容當作已知，不必多做繁瑣的證明。

- Let  $\beta_1, \dots, \beta_3$  be the angles for a triangle in degrees, so  $\beta_1 + \beta_2 + \beta_3 = 180$ ; and let  $Y_1, \dots, Y_3$  be measurements of these angles. Assume  $Y_i \sim N(\beta_i, \sigma^2)$ ,  $i=1,2,3$ .
  - Find the MLE for  $\beta_1$  and  $\beta_2$ .
  - Find a unbiased estimator of  $\sigma^2$ .
- Let  $X$  and  $Y$  be independent r.v.'s,  $X \sim \text{Exp}(1)$ , and  $Y \sim U(0,1)$ . Let  $V=X/(X+Y)$ .
  - Find  $P(V>c|Y=y)$  for  $0<c<1$ .
  - Compute  $P(V>c)$ .
  - What is the density of  $V$ ?
- Let  $X$  and  $Y$  be i.i.d. r.v.'s with  $\text{Exp}(1)$  distribution. Let  $Z=X/Y$ .
  - For  $z>0$ , find  $P(Z<z|Y=y)$ .
  - Compute  $P(Z<z)$ ,  $z>0$ .
- Let  $X$  and  $Y$  be r.v.'s with joint density  $p(x,y)=2y^2\exp(-xy)$ ,  $x>0$ ,  $0<y<1$ .
  - Find the marginal density for  $Y$ .
  - Find the conditional density for  $X$  given  $Y=y$ .
  - Find  $P(X>1|Y=y)$ ,  $E[X|Y=y]$ , and  $E[X^2|Y=y]$ .
- Suppose  $X$  has a Poisson distribution with mean  $\lambda$  and that given  $X=x$ ,  $Y$  has a binomial distribution with  $x$  trials and success probability  $p$ .
  - Find the marginal density for  $Y$ .
  - Find the conditional density for  $X$  given  $Y=y$ .
  - Find  $E[Y^2|X]$ .
  - Compute  $EY^2$ .
- Suppose  $X \sim N(0,1)$  and  $Y|X=x \sim N(x,1)$ .
  - Find the mean and variance of  $Y$ .
  - Find the conditional density for  $X$  given  $Y=y$ .